Modeling Pursuit Curves Using Differential Equations  
Carlota Bonnet, Cesar Garcia, and Mitchell Herold  
Spring 2017

Introduction
A pursuit curve is a curve showing the path an object takes as it pursues another object. In real life, it may be a hound chasing a rabbit, an outfielder running towards a pop-fly, or a guided missile pursuing a jet. Graphically, the velocity vector of the pursuer is aimed directly towards prey (on the prey’s tangent), and therefore is a straight line. Through this project, we will explain how to model pursuit curves using differential equations.

Assumptions
- Object A is pursued by object B
- Both objects are moving at a constant speed and will continue to do so throughout any interval of time (endurance is ignored)
- There is a constant of proportionality relating their speeds, $k$, such that $k \|v_A\| = \|v_B\|$
- Both objects can turn at any rate dictated by their functions of movement (mass and agility are non-issues)
- To allow the chaser to always be pointed at its target, we give it a reaction time of zero to the target’s course change
- We are given the functions of the pursued object, the starting positions (initial conditions) of both objects, and $k$

Equations
Pursued curve: $\hat{A}(t) = p(t)\hat{i} + q(t)\hat{j}$
Pursuit curve: $\hat{B}(t) = x(t)\hat{i} + y(t)\hat{j}$

Straight Trajectory
Example: A fox pursuing a rabbit running in a straight line
\[
\frac{dx}{dt} = k\left[ \frac{-x}{\sqrt{(-x)^2 + (t-y)^2}} \right]
\]
\[
\frac{dy}{dt} = k\left[ \frac{t-y}{\sqrt{(-x)^2 + (t-y)^2}} \right]
\]

Circular Trajectory
Example: A dog pursuing a duck going around in circles in a pond
\[
\frac{dx}{dt} = k \left[ \frac{(p-x)}{\sqrt{(p-x)^2 + (q-y)^2}} \right]
\]
\[
\frac{dy}{dt} = k \left[ \frac{(q-y)}{\sqrt{(p-x)^2 + (q-y)^2}} \right]
\]

The Mice Problem
We suppose a mouse is placed on each vertex of an n-polygon, and each mouse is pursuing the mouse on its right. Then, the mice’s trajectories create a logarithmic spiral and meet at the center of the polygon.

These spirals are often used in art.

References
Arnold, Dave & P. Gent. "Pursuit Curves and MatLab." College of the Redwoods online.
Professor Aharon Dagan, Santa Fe College
Therrien, Edward. "Pursuit Curves."