Modeling Pursuit Curves Using Differential Equations

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Introduction

A pursuit curve is a curve showing the path an object takes as it pursues another object. In real life, it may be a hound chasing a rabbit, an outfielder running towards a pop-fly, or a guided missile pursuing a jet. Graphically, the velocity vector of the pursuer is aimed directly towards prey (on the prey's tangent), and therefore is a straight line. Through this project, we will explain how to model pursuit curves using differential equations.

Assumptions

> Object A is pursued by object B
> Both objects are moving at a constant speed and will continue to do so throughout any interval of time (endurance is ignored)
> There is a constant of proportionality relating their speeds, *k,* such that k‖Va‖= ‖Vb‖.
> Both objects can turn at any rate dictated by their functions of movement (mass and agility are non-issues)
> To allow the chaser to *always* be pointed at its target, we give it a reaction time of zero to the target's course change
> We are given the functions of the pursued object, the starting positions (initial conditions) of both objects, and *k*

Equations

Pursued curve: $\vec{A}(t) = p(t)\hat{i} + q(t)\hat{j}$

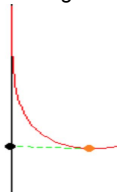
Pursuit curve: $\vec{B}(t) = x(t)\hat{i} + y(t)\hat{j}$

Straight Trajectory

Example: A fox pursuing a rabbit running in a straight line

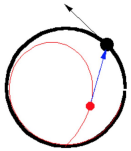
$$\frac{dx}{dt} = k\left[\frac{-x}{\sqrt{(-x)^2+(t-y)^2}}\right]$$

$$\frac{dy}{dt} = k\left[\frac{t-y}{\sqrt{(-x)^2+(t-y)^2}}\right]$$



Circular Trajectory

Example: A dog pursuing a duck going around in circles in a pond

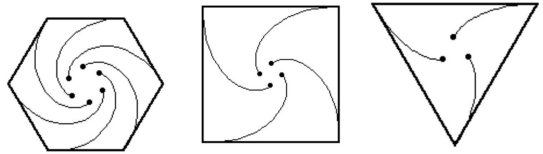


$$\frac{dx}{dt} = k\left[\frac{(p-x)}{\sqrt{(p-x)^2+(q-y)^2}}\right]$$

$$\frac{dy}{dt} = k\left[\frac{(q-y)}{\sqrt{(p-x)^2+(q-y)^2}}\right]$$

The Mice Problem

We suppose a mouse is placed on each vertex of an n-polygon, and each mouse is pursuing the mouse on its right. Then, the mice's trajectories create a logarithmic spiral and meet at the center of the polygon.



These spirals are often used in art.

References

Arnold, Dave & P. Gent. "Pursuit Curves and MatLab." College of the Redwoods online.
Floyd, Michael. (2006) "Pursuit Curves." *Academic Forum*.
Professor Aharon Dagan, Santa Fe College
Nahin, Paul J. (2007). *Chases and Escapes: The Mathematics of Pursuit and Evasion.* Princeton, New Jersey: Princeton University Press.
Stewart, James. (2003). *Calculus: Early Transcendentals*, 5th ed. Belmont, California: Brooks/Cole-Thomson Learning.
Therrien, Dan. n.d. "Pursuit Curves".