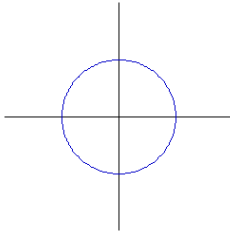


These study questions are intended to offer the student the opportunity to practice problems covered in MAC1105 and should serve as a general review before taking the final exam. These are not the exam questions. The number of study questions per topic does not necessarily reflect the distribution of problems that will appear on the final exam.

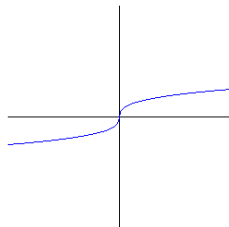
The common final exam for College Algebra consists of multiple choice and free response questions. Students will not be given partial credit for the multiple choice questions; however, students may earn partial credit for the free response questions. NO books, NO formula sheets, and NO notes are allowed during testing!

1. Determine whether each set of points represents a function.
 - a. $(2,6), (-4,6), (1,-3), (4,-3)$
 - b. $(2,-2), (3,-2), (4,-2), (6,-2)$
 - c. $(2,-3), (2,3), (-2,-3), (-2,3)$
 - d. $(-1,2), (-1,0), (-1,-1), (-1,-2)$

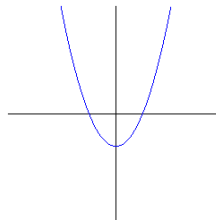
2. Which of the accompanying graphs describe functions? Explain your answer.



Graph A



Graph B



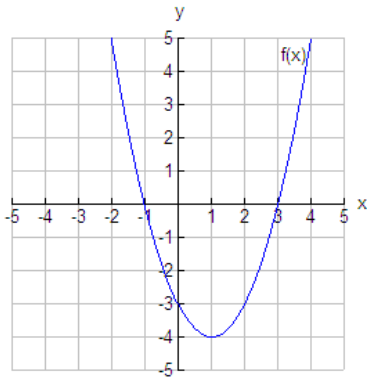
Graph C

3. The cost of driving a car to work is estimated to be \$2.00 in tolls plus 32 cents per mile. Write an equation for computing the total cost C of driving M miles to work. Does your equation represent a function? What is the independent variable? What is the dependant variable? Generate a table of values and then graph the equation.

4. Look at the accompanying table.
 - a. Find $p(-4)$, $p(5)$, and $p(1)$.
 - b. For what value(s) of n does $p(n)=2$?

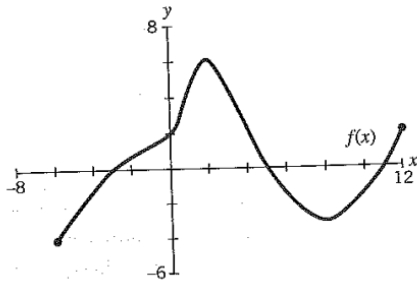
n	-4	-3	-2	-1	0	1	2	3	4	5
$p(n)$	0.063	0.125	0.25	0.5	1	2	4	8	16	32

5. From the accompanying graph of $y = f(x)$:



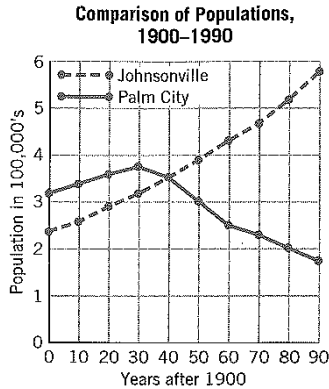
- Find $f(-2)$, $f(-1)$, $f(0)$, and $f(1)$.
- Find two values of x for which $f(x) = -3$.
- Estimate the range of f . Assume that the arms of the graph extend upward indefinitely.

6. Use the graph below to answer the following questions about $f(x)$.



- Over which interval(s) is $f(x) < 0$?
- Over which interval(s) is $f(x) > 0$?
- Over which interval(s) is $f(x)$ increasing?
- Over which interval(s) is $f(x)$ decreasing?
- How would you describe the concavity of $f(x)$ over the interval $(0, 5)$ for x ? Over $(5, 8)$ for x ?
- Find a value for x when $f(x) = 4$.
- $f(-8) = ?$

7. Examine the accompanying graph, which shows the populations of two towns.



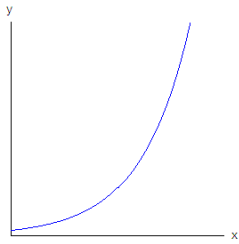
- a. What is the range of population size for Johnsonville? For Palm City?
 - b. During what years did the population of Palm City increase?
 - c. During what years did the population of Palm City decrease?
 - d. When were the populations equal?
8. (Graphing program required.) Use technology to graph the following functions and then complete both sentences for each function.
- $y_1 = x^3$ $y_2 = x^2$ $y_3 = \frac{1}{x+3}$ $y_4 = \frac{1}{x} + 2$
- a. As x approaches positive infinity, y approaches _____.
 - b. As x approaches negative infinity, y approaches _____.
9. Though reliable data about the number of African elephants are hard to come by, it is estimated that there were about 4,000,000 in 1930 and only 500,000 in 2000.
- a. What is the average annual rate of change in elephants over time? Interpret your result.
 - b. During the 1980's it was estimated that 100,000 elephants were being killed each year due to systematic poaching for ivory. How does this compare with your answer in part (a)?
What does this tell you about what was happening before or after the 1980's?
10. Calculate the average rate of change between adjacent points for the following function. (The first few are done for you.)

x	$f(x)$	Average Rate of Change
0	0	n.a.
1	1	1
2	8	7
3	27	
4	64	
5	125	

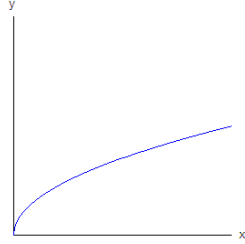
- a. Is the function $f(x)$ increasing, decreasing, or constant throughout?
- b. Is the average rate of change increasing, decreasing, or constant throughout?

11. Each of the following functions has a graph that is increasing. If you calculated the average rate of change between sequential equal-size intervals, which function can be said to have an average rate of change that is:

- a. Constant? b. Increasing? c. Decreasing?



Graph A



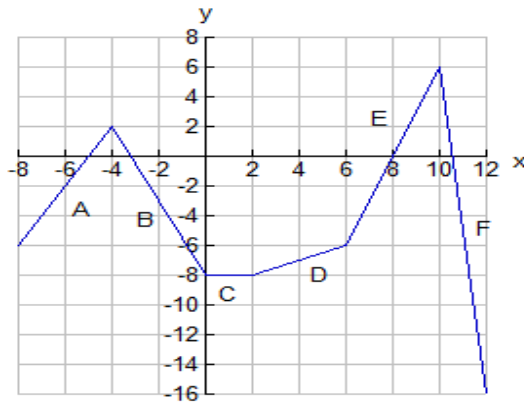
Graph B



Graph C

12. Given the graph below:

- a. Estimate the slope for each line segment A-F.
 b. Which line segment is the steepest?
 c. Which line segment has a slope of zero?



13. Handicapped Vietnam veterans successfully lobbied for improvements in the architectural standards for wheelchair access in public buildings.

- a. The old standard was 1-foot rise for every 10 horizontal feet. What would the slope be for a ramp built under this standard?
 b. The new standard is 1-foot rise for every 12 horizontal feet. What would the slope of the ramp be under this standard?
 c. If the front door is 3 feet above the ground, how long would the handicapped ramp be, using the old standard? Using the new?

14. Consider the equation $D = 3.40 + 0.11n$.

- a. Find the values of D for $n = 0, 1, 2, 3, 4$.
 b. If D represents the average consumer debt, in thousands of dollars, over n years, what does 0.11 represent? What are its units?

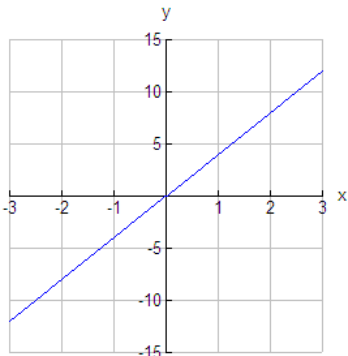
15. Match the graph with the correct equation.

a. $y = x$

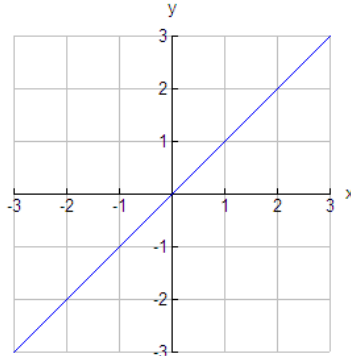
b. $y = 2x$

c. $y = \frac{x}{2}$

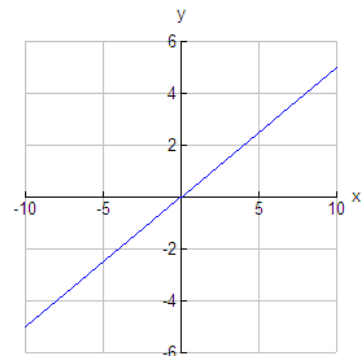
d. $y = 4x$



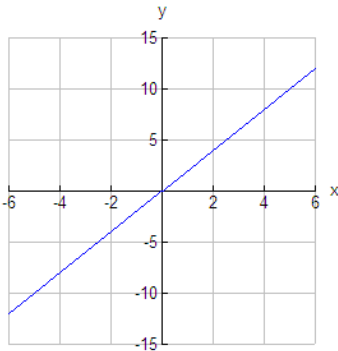
Graph A



Graph B



Graph C



Graph D

16. A teacher's union has negotiated a uniform salary increase for each year of service up to 20 years. If a teacher started at \$26,000 and 4 years later had a salary of \$32,000:

- What was the annual increase?
- What function would describe the teacher's salary over time?
- What would be the domain for the function?

17. In 1977 a math professor bought her condominium in Cambridge, Massachusetts, for \$70,000. The value of the condo has risen steadily so that in 2007 real estate agents tell her the condo is now worth \$850,000.

- Find a formula to represent these facts about the value of the condo $V(t)$, as a function of time, t .
- If she retires in 2010, what does your formula predict her condo will be worth then?

18. Find the equation of the line in form $y = mx + b$ for each of the following sets of conditions. Show your work.

- Slope is \$1400/year and line passes through the point (10 yr, \$12,000).
- Line is parallel to $2y - 7x = y + 4$ and passes through the point (-1,2).
- Equation is $1.48x - 2.00y + 4.36 = 0$.

- d. Line is horizontal and passes through (1.0, 72.).
- e. Line is vertical and passes through (275, 1029).
- f. Line is perpendicular to $y = -2x + 7$ and passes through (5,2).

19. The Gas Guzzler Tax is imposed on manufacturers on the sale of new-model cars (not minivans, sport utility vehicles, or pickup trucks) whose fuel economy fails to meet certain statutory regulations, to discourage the production of fuel-inefficient vehicles. The tax is collected by the IRS and paid by the manufacturer. The table shows the amount of tax that the manufacturer must pay for a vehicle's miles per gallon fuel efficiency.

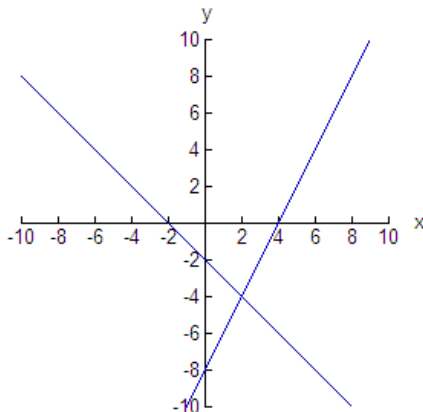
Gas Guzzler Tax

MPG	Tax Per Car
12.5	\$6400
13.5	\$5400
14.5	\$4500
15.5	\$3700
16.5	\$3700
17.5	\$2600
18.5	\$2100
19.5	\$1700
20.5	\$1300
21.5	\$1000
22.5	\$0

(Source: <http://www.epa.gov/otaq/cert/factsheets/fefact o.1. pdf>)

- a. Plot the data, verify that they are roughly linear, and add a line of best fit.
- b. Choose two points on the line, find the slope, and then form a linear equation with x as the fuel efficiency in mpg and y as the tax in dollars.
- c. What is the rate of change of the amount of tax imposed on fuel-inefficient vehicles? Interpret the units.

20. Create the system of equations that produced the accompanying graph. Estimate the solution for the system from the graph and then confirm using your equations.



21. The supply and demand equations for a particular bicycle model relate price per bicycle, p (in dollars) and q , the number of units (in thousands). The two equations are:

$$p = 250 + 40q \quad \text{Supply}$$

$$p = 510 - 25q \quad \text{Demand}$$

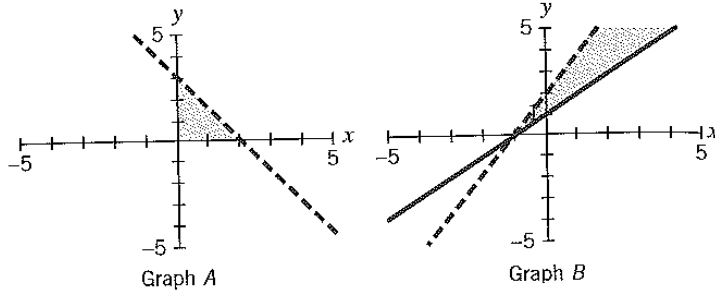
- Sketch the equations on the same graph. On your graph identify the supply equation and the demand equation.
 - Find the equilibrium point and interpret its meaning.
22. While totally solar energy-powered home energy systems are quite expensive to install, passive solar systems are much more economical. Many passive solar features can be incorporated at the time of construction with a small additional initial cost to a conventional system. These features enable energy costs to be one-half to one-third of the costs in conventional homes.

Below is the cost analysis from the case study Esperanza del Sol.

Cost of installation of a conventional system:	\$10,000
Additional cost to install passive solar features:	\$150
Annual energy costs for conventional system:	\$740
Annual energy costs of hybrid system with Additional passive solar features:	\$540

- Write the cost equation for the conventional system.
 - Write the cost equation for the passive hybrid solar system.
 - When would the total cost of the passive hybrid solar system be the same as the conventional system?
 - After 5 years, what would be the total energy cost of the passive hybrid solar system? What would be the total cost of the conventional system?
- (Adapted from Buildings for a Sustainable America: Case Studies, American Solar Energy Society, Boulder, CO.)*
23. Two professors from Purdue University reported that for a typical small-sized fertilizer plant in Indiana the fixed costs were \$235,487 and it cost \$206.68 to produce each ton of fertilizer.
- If the company planned to sell the fertilizer at \$266.67 per ton, find the cost, C , and revenue, R , equations for x tons of fertilizer.
 - Graph the cost and revenue equations on the same graph and calculate and interpret the breakeven point.
 - Indicate the region where the company would make a profit.

24. Describe the shaded region in each graph with the appropriate inequalities.



25. Without using a calculator, solve each equation for x .

- a. $10^{x-5} = 1000$ c. $10^{3x-1} = 0.0001$
 b. $\log(2x + 10) = 2$ d. $\log(500 - 25x) = 3$

26. A cancer patient's white blood cell count grew exponentially after she had completed chemotherapy treatments. The equation $C = 63(1.17)^d$ describes C , her white blood cell count per milliliter, d days after the treatment was completed.

- What is the white blood cell count growth factor?
- What was the initial white blood cell count?
- Create a table of values that shows the white blood cell counts from day 0 to day 10 after the chemotherapy.
- From the table of values, approximate when the number of white blood cells doubled.

27. A tuberculosis culture increases by a factor of 1.185 each hour.

- If the initial concentration is $5 \cdot 10^3$ cells/ml, construct an exponential function to describe its growth over time.
- What will the concentration be after 8 hours?

28. A small village has an initial size of 50 people at a time $t=0$, with t in years.

- If the population increases by 5 people per year, find the formula for the population size $P(t)$.
- If the population increases by a factor of 1.05 per year, find a new formula $Q(t)$ for the population size.
- Plot both functions on the same graph over a 30-year period.
- Use your graphing calculator to determine the coordinates of the point(s) where the graphs intersect. Interpret the meaning of the intersection point(s).

29. Each of the following tables contains values representing either linear or exponential functions. Find the equation for each function.

a.

x	-2	-1	0	1	2
F(x)	1.12	2.8	7	17.5	43.75

b.

x	-2	-1	0	1	2
G(x)	0.1	0.3	0.5	0.7	0.9

30. Write an equation for an exponential decay function where:

- The initial population is 10,000 and the decay factor is $\frac{2}{5}$.
- The initial population is $2.7 \cdot 10^{13}$ and the decay factor is 0.27.

31. Plutonium -238 is used in bombs and power plants but is dangerously radioactive. It decays very slowly into nonradioactive materials. If you started with 100 grams today, a year from now you would have 99.2 grams.

- Construct an exponential function to describe the decay of plutonium-238 over time.
- How much of the original 100 grams of plutonium-238 would be left after 50 years?
After 500 years?

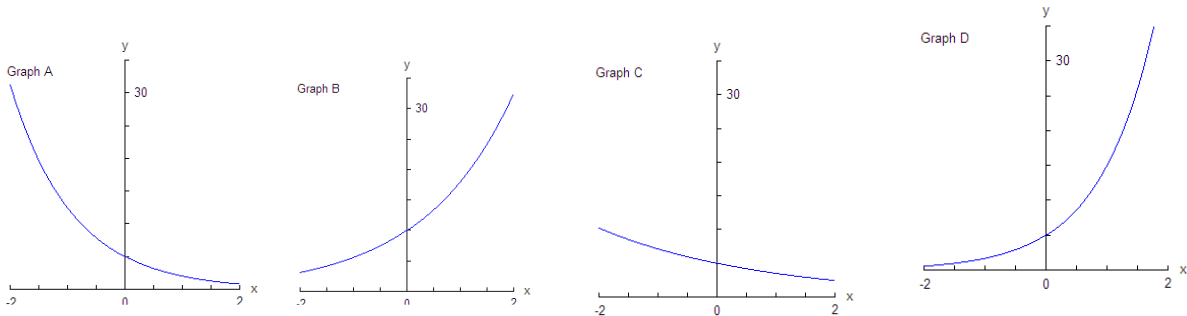
32. Below are graphs of four exponential functions. Match each function with its graph.

$$P = 5 \cdot (0.7)^X$$

$$R = 10 \cdot (1.8)^X$$

$$Q = 5 \cdot (0.4)^X$$

$$S = 5 \cdot (3)^X$$



33. Graph the functions $f(x) = 30 + 5x$ and $g(x) = 3(1.6)^X$ on the same grid. Supply the symbol < or > in the blank that would make the statement true.

- $f(0)$ _____ $g(0)$
- $f(6)$ _____ $g(6)$
- $f(7)$ _____ $g(7)$
- $f(-5)$ _____ $g(-5)$
- $f(-6)$ _____ $g(-6)$
- As $x \rightarrow +\infty$, $f(x)$ _____ $g(x)$
- As $x \rightarrow -\infty$, $f(x)$ _____ $g(x)$

34. Fill in the following chart and then construct exponential functions for each part (a) to (g).

	Initial Value	Growth or Decay?	Growth or Decay Factor	Growth or Decay Rate (% form)	Exponential Function
a.	600		2.06		
b.	1200			200%	
c.	6000	Decay		75%	
d.	1,500,000	Decay		25%	
e.	1,500,000	Growth		25%	
f.	7		4.35		
g.	60		0.35		

35. The table below represents an exponential function. Construct that function and then identify the corresponding growth or decay rate in percentage form.

X	0	1	2	3
Y	500.00	425.00	361.25	307.06

36. Generate equations that represent the pollution levels, $P(t)$, as a function of time, t (in years), such that $P(0) = 150$ and:

- $P(t)$ triples each year.
- $P(t)$ decreases by twelve units each year.
- $P(t)$ decreases by 7% each year.
- The annual average rate of change of $P(t)$ with respect to t is constant at 1.

37. A pollutant was dumped into a lake, and each year its amount in the lake is reduced by 25%.

- Construct a general formula to describe the amount of pollutant after t years if the original amount is A_0 .
- Use a symbolic approach to determine how long it will take before the pollution is reduced to 10% of its original level.

38. Which of the following functions have a fixed doubling time? A fixed half-life?

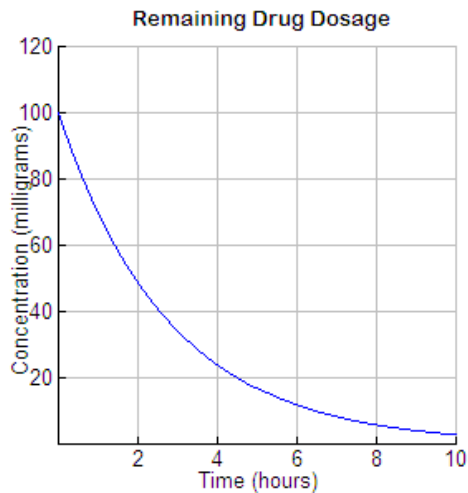
- $Y = 6(2)^x$
- $Y = 5 + 2x$
- $Q = 300(1/2)^t$
- $A = 10(2)^{\frac{t}{5}}$
- $P = 500 - \frac{1}{2}T$
- $N = 50\left(\frac{1}{2}\right)^{t/20}$

39. Fill in the following chart. (The first column is done for you.)

	a.	b.	c.	d.
Initial Value	50	1000	4	5000
Doubling Value	30 days	7 years	25 minutes	18 months
Exponential Function ($f(t) = Ca^{t/n}$)	$F(t) = 50 (2)^{t/30}$			
Growth Factor Per unit of time	$2^{1/30} = 1.0234$ Per day			
Growth rate per Unit of time (in percentage form)	2.34% Per day			

40. The accompanying graph shows the concentration of drug in the human body as the initial amount of 100 mg dissipates over time. Estimate when the concentration becomes:

- a. 60 mg b. 40 mg c. 20 mg



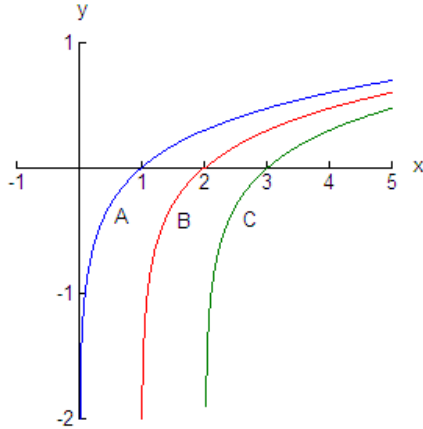
41. Contract, using the rules of logarithms, and express your answer as a single logarithm.

- a. $3 \log K - 2 \log(K+3)$
 b. $-\log m + 5 \log(3+n)$
 c. $4 \log T + \frac{1}{2} \log T$
 d. $\frac{1}{3} (\log x + 2 \log y)$

42. Solve for x.

- a. $\log x = 3$ d. $\log(x + 1) - \log x = 1$
 b. $\log(x + 1) = 3$ e. $\log x - \log(x + 1) = 1$
 c. $3 \log x = 5$

43. The function $N = N_0 \cdot 1.5^t$ describes the actual number N of E. coli bacteria in an experiment after t time periods (of 20 minutes each) starting with an initial bacteria count of N_0 .
- What is the doubling time?
 - How long would it take for there to be ten times the original number of bacteria?
44. The half-life of bismuth-214 is about 20 minutes.
- Construct a function to model the decay of bismuth-214 over time. Be sure to specify your variables and their units.
 - For any given sample of bismuth-214, how much is left after 1 hour?
 - How long will it take to reduce the sample to 25% of its original size?
 - How long will it take to reduce the sample to 10% of its original size?
45. Construct functions for parts (a) and (b) and compare them in parts (c) and (d).
- \$25,000 is invested at 5.75% compounded quarterly.
 - \$25,000 is invested at 5.75% compounded continuously.
 - What is the amount in each account at the end of 5 years?
 - Determine the effective rate of interest for each, then explain why one amount is larger than the other.
46. Uranium-238 decays continuously at a rate of about 13.9% every billion years. Assume you start with 10 grams of U-238.
- Construct an equation to describe the amount of U-238 remaining after x billion years.
 - How long would it take for 10 grams of U-238 to become 5 grams?
47. Use rules of logarithms to expand.
- $\ln(\sqrt{4xy})$
 - $\ln\left(\frac{\sqrt[3]{2x}}{4}\right)$
 - $\ln(3\sqrt[4]{x^3})$
48. Solve for x .
- $e^x = 10$
 - $2 + 4^x = 7$
 - $\ln(x + 1) = 3$
 - $10^x = 3$
 - $\ln x = 5$
 - $\ln x - \ln(x + 1) = 4$
49. The interest on an account is compounded continuously at 7%. Estimate the effective interest rate.
50. The functions $f(x) = \log x$, $g(x) = \log(x - 1)$, and $h(x) = \log(x - 2)$ are graphed below.
- Match each function with its graph.
 - Find the value of x for each function that makes that function equal to zero.
 - Identify the x -intercept for each function.



51. Rewrite each of the following functions using base e .

- a. $N = 10(1.045)^t$ c. $P = 500(2.10)^x$
 b. $Q = (5 \cdot 10^{-7}) \cdot (0.072)^A$

52. After t days, the amount of thorium-234 in a sample is $A(t) = 35e^{-0.029t}$ micrograms.

- How much was there initially?
- How much is there after a week?
- When is there just 1 microgram left?
- What is the half-life of thorium-234?

53. Radioactive thorium-230 decays according to the formula $Q = Q_0\left(\frac{1}{2}\right)^{t/75000}$, where Q_0 is the initial quantity in milligrams and t is measured in years.

- What is the half-life of thorium-230?
- Translate the equation into the form $Q = Q_0a^t$.
- What is the annual decay rate?

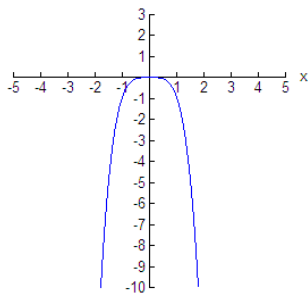
54. In 1859, the Victorian landowner Thomas Austin imported 12 wild rabbits into Australia and let them loose to breed. Since they had no natural enemies, the population increased very rapidly. By 1949 there were approximately 600 million rabbits.

- Find an exponential function using a continuous growth rate to model this situation.
- If the growth had gone unchecked, what would have been the rabbit population in 2000?

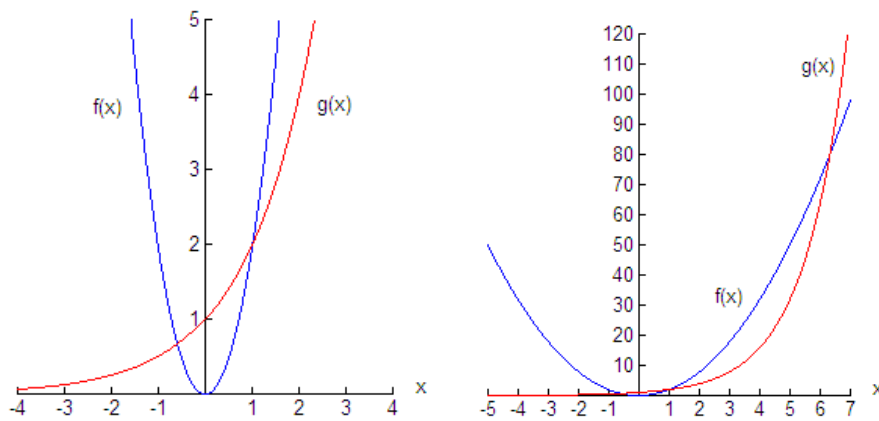
55. If the radius of a sphere is x meters, surface area $= 4\pi r^2$ and volume $= \frac{4}{3}\pi r^3$, what happens to the surface area S and to the volume V of a sphere when you:

- Quadruple the radius?
- Multiply the radius by n ?
- Divide the radius by 3?
- Divide the radius by n ?

56. Assume Y is directly proportional to X^3 .
- Express this relationship as a function where Y is the dependent variable.
 - If $Y = 10$ when $X = 2$, then find the value of the constant of proportionality in part (a).
 - If X is increased by a factor of 5, what happens to the value of Y ?
57. Assume L is directly proportional to x^5 . What is the effect of doubling x ?
58. Write a general formula to describe each variation. Use the information given to find the constant of proportionality.
- Q is directly proportional to the product of the cube root of t and the square of d , and $Q = 18$ when $t = 8$ and $d = 3$.
 - A is directly proportional to the product of the height, h , and the square of the radius, r , and $A = 100\pi$ when $r = 5$ and $h = 2$.
 - V is directly proportional to the product of B and h , and $V = 192$ when $B = 48$ and $h = 4$.
 - T is directly proportional to the product of the square root of p and the square of u , and $T = 18$, when $p = 4$ and $u = 6$.
59. Plot the functions $f(x) = x^2$ and $g(x) = x^5$ on the same grid. Insert the symbol $>$, $<$, or $=$ to make the relation true.
- For $x = 0$, $f(x)$ _____ $g(x)$
 - For $0 < x < 1$, $f(x)$ _____ $g(x)$
 - For $x = 1$, $f(x)$ _____ $g(x)$
 - For $x > 1$, $f(x)$ _____ $g(x)$
 - For $x < 0$, $f(x)$ _____ $g(x)$
60. Consider the accompanying graph of $f(x) = kx^n$, where n is a positive integer.
- Is n even or odd?
 - Is $k > 0$ or is $k < 0$?
 - Does $f(-2) = f(2)$?
 - Is $f(-x) = f(x)$?
 - As $x \rightarrow +\infty$, $f(x) \rightarrow$ _____
 - As $x \rightarrow -\infty$, $f(x) \rightarrow$ _____



61. The following figures show two different versions of the graphs of $f(x) = 2x^2$ and $g(x) = 2^x$. Notice that each version uses different scales on the axes. Make sure to check each version when answering the following questions.

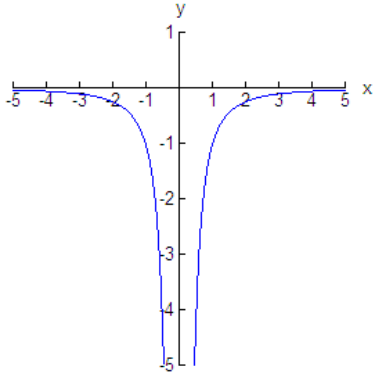


Supply the appropriate inequality symbol.

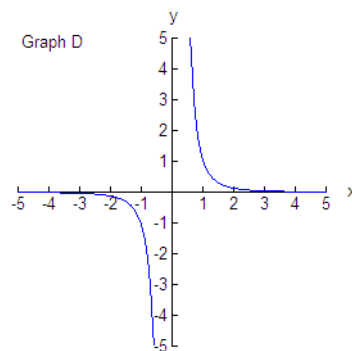
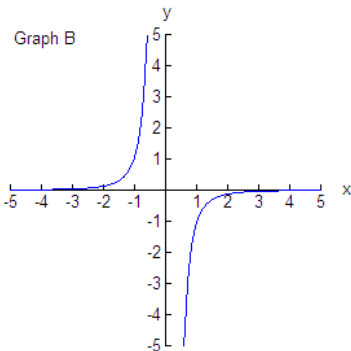
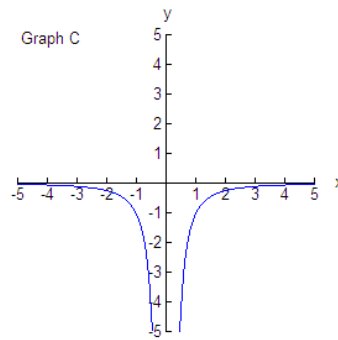
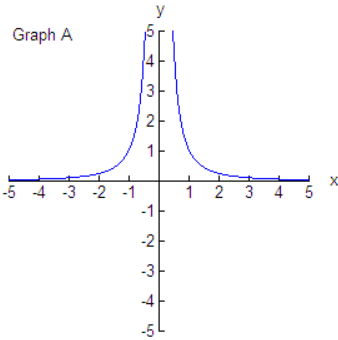
- As $x \rightarrow -\infty$, $f(x)$ _____ $g(x)$
 - As $x \rightarrow +\infty$, $f(x)$ _____ $g(x)$
 - When x lies in the interval $(-\infty, -1)$, then $f(x)$ _____ $g(x)$.
 - When x lies in the interval $(-0.5, 1)$, then $f(x)$ _____ $g(x)$.
 - When x lies in the interval $(1, 6)$, then $f(x)$ _____ $g(x)$.
 - When x lies in the interval $(7, \infty)$, then $f(x)$ _____ $g(x)$.
62. Find power, exponential, and linear equations that go through the two points $(1, 0.5)$ and $(4, 32)$.
63. The intensity of light from a point source is inversely proportional to the square of the distance from the light source. If the intensity is 4 watts per square meter at a distance of 6 m from the source, find the intensity at a distance of 8 m from the source. Find the intensity at a distance of 100 m from the source.
64. Boyle's Law says that if the temperature is held constant, then the volume, V , of a fixed quantity of gas is inversely proportional to the pressure, P . That is, $V = \frac{k}{p}$ for some constant k . What happens to volume if:
- The pressure triples?
 - The pressure is multiplied by n ?
 - The pressure is halved?
 - The pressure is divided by n ?

65. Consider the accompanying graph of $f(x) = k \cdot \frac{1}{x^n}$, where n is a positive integer.

- Is n even or odd?
- Is $k > 0$ or is $k < 0$?
- Is $f(-1) > 0$ or is $f(-1) < 0$?
- Does $f(-x) = f(x)$?
- As $x \rightarrow +\infty$, $f(x) \rightarrow$ _____
- As $x \rightarrow -\infty$, $f(x) \rightarrow$ _____



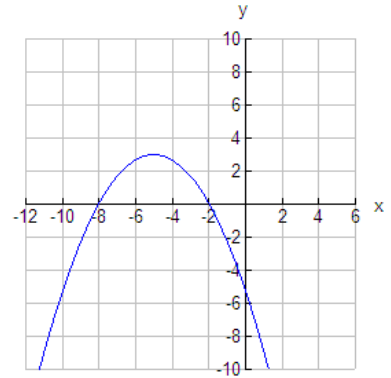
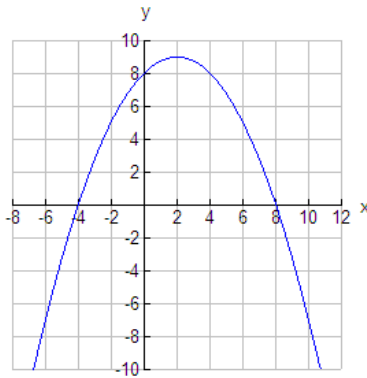
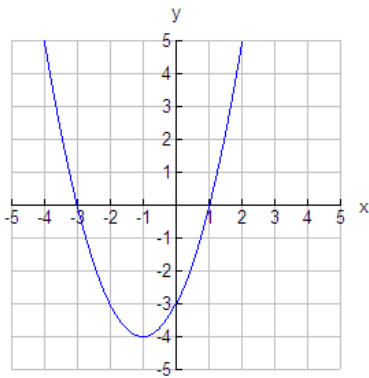
66. Consider the following graphs of four functions of the form $y = \frac{k}{x^n}$, where $n > 0$.



Which functions:

- Are symmetric across the y-axis?
- Are symmetric about the origin?
- Have an even power n ?
- Have an odd power n ?
- Have $k < 0$?

67. From the graph of each quadratic function, identify whether the parabola is concave up or down and hence whether the function has a maximum or minimum. Then estimate the vertex, the axis of symmetry, and any horizontal and vertical intercepts.



68. Find the coordinates of the vertex for each quadratic function listed. Then specify whether each vertex is a maximum or minimum.

a. $y = 4x^2$

b. $P(n) = \left(\frac{1}{12}\right)n^2$

c. $f(x) = -8x^2$

d. $Q(t) = -\left(\frac{1}{12}\right)t^2$

69. For each of the following quadratic functions, find the vertex (h, k) and determine if it represents the maximum or minimum of the function.

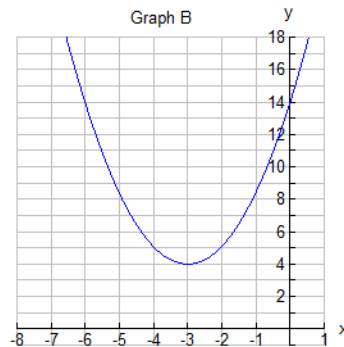
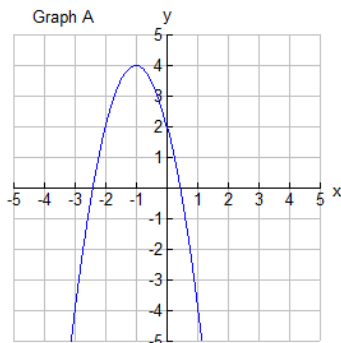
a. $f(x) = -2(x - 3)^2 + 5$

c. $f(x) = -5(x + 4)^2 - 7$

b. $f(x) = 1.6(x + 1)^2 + 8$

d. $f(x) = 8(x - 2)^2 - 6$

70. Construct an equation for each of the accompanying parabolas.



71. Marketing research by a company has shown that the profit, $P(x)$ (in thousands of dollars), made by the company is related to the amount spent on advertising, x (in thousands of dollars), by the equation $P(x) = 230 + 20x - 0.5x^2$. What expenditure (in thousands of dollars) for advertising gives the maximum profit? What is the maximum profit?

72. The following function represents the relationship between time t (in seconds) and height h (in feet) for objects thrown upward on Pluto. For an initial velocity of 20 ft/sec and an initial height above the ground of 25 feet, we get:

$$h = -t^2 + 20t + 25$$

- Find the coordinates of the point where the graph intersects the h -axis, then interpret this point.
- Find the coordinates of the vertex of the parabola.
- Sketch the graph. Label the axes.
- Interpret the vertex in terms of time and height.
- For what values of t does the mathematical model make sense?

73. Calculate the coordinates of the x - and y -intercepts for the following quadratics. Give each intercept as an ordered pair.

a. $Y = 3x^2 + 2x - 1$

c. $y = (5 - 2x)(3 + 5x)$

b. $Y = 3(x - 2)^2 - 1$

d. $f(x) = x^2 - 5$

e. $f(x) = x^2 + x + 1$

74. Use the discriminant to predict the number of horizontal intercepts for each function. Then use the quadratic formula to find any horizontal intercepts.

a. $Y = 2x^2 + 3x - 5$

b. $F(x) = -16 + 8x - x^2$

c. $F(x) = x^2 + 2x + 2$

d. $Y = 2(x - 1)^2 + 1$

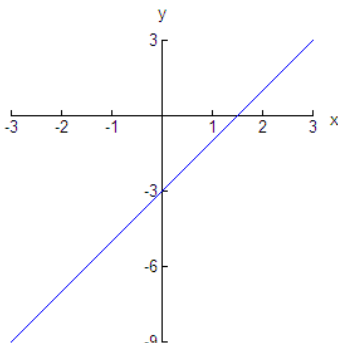
e. $G(z) = 5 - 3z - z^2$

75. Match each of the following functions with its graph.

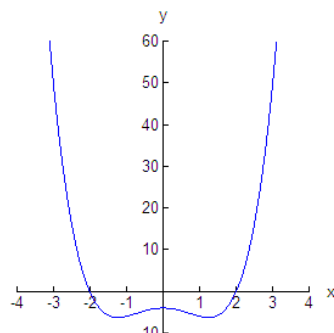
a. $y = 2x - 3$

b. $y = 3(2^x)$

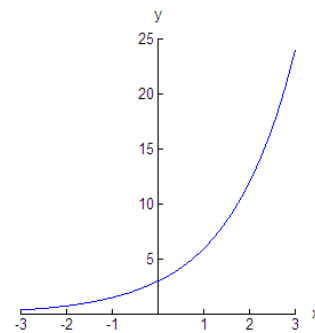
c. $y = (x^2 + 1)(x^2 - 4)$



Graph A



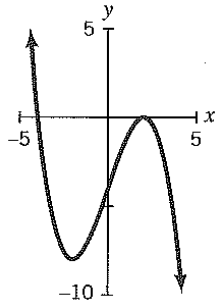
Graph B



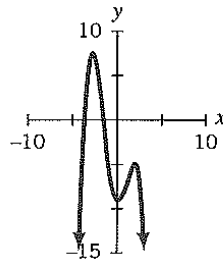
Graph C

76. For each of the following graphs of polynomial functions, determine (assuming the arms extend indefinitely in the indicated direction):

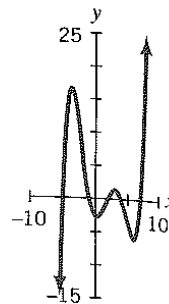
- The number of turning points
- The number of x-intercepts
- The sign of the leading term
- The minimum degree of the polynomial



Graph A



Graph B



Graph C

77. In each part, construct a polynomial function with the indicated characteristics.

- Crosses the x-axis at least three times
- Crosses the x-axis at -1 , 3 , and 10
- Has a y-intercept of 4 and degree of 3
- Has a y-intercept of -4 and degree of 5

78. Which of the following statements are true about the graph of the polynomial function:

$$F(x) = x^3 + bx^2 + cx + d$$

- It intersects the vertical axis at one and only one point.
- It intersects the x-axis in at most three points.
- It intersects the x-axis at least once.
- The vertical intercept is positive.
- As $x \rightarrow \infty, F(x) \rightarrow \infty$.
- It has at most two turns.

79. Let $f(x) = 3x^5 + x$ and $g(x) = x^2 - 1$.

- Construct the following functions.
 $j(x) = f(x) + g(x)$, $k(x) = f(x) - g(x)$, $m(x) = f(x) \cdot g(x)$
- Evaluate: $j(2)$, $k(3)$, and $m(-1)$.

80. Using the accompanying table, evaluate the following expressions in parts (a) – (d).

x	0	1	2	3	4	5
f(x)	-3	-1	5	15	29	47

X	0	1	2	3	4	5
g(x)	-3	-5	-11	-21	-35	-53

- a. $(f + g)(2)$ b. $(g - f)(2)$ c. $(f \cdot g)(3)$ d. $\left(\frac{g}{f}\right)(1)$

81. For the following function, identify the any horizontal and/or vertical asymptotes. Then, if possible, use technology to graph the function and verify your results.

$$f(x) = \frac{3x-13}{x-4}$$

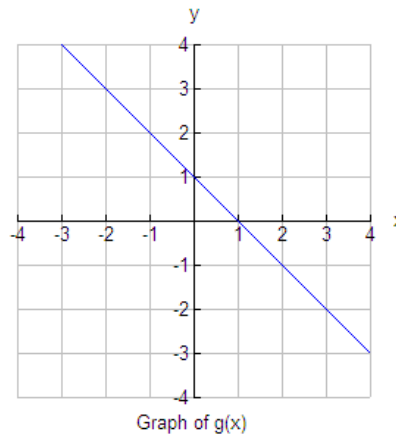
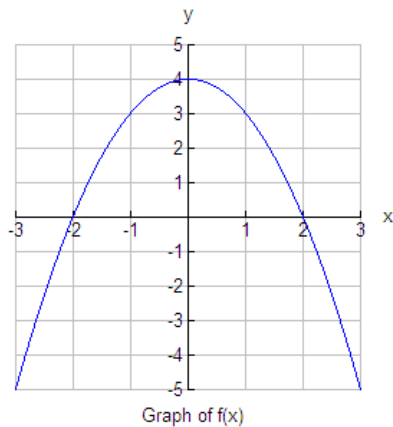
82. From the accompanying table, find:

- a. $f(g(1))$ b. $g(f(1))$ c. $f(g(0))$ d. $g(f(0))$ e. $f(f(2))$

X	f(x)	g(x)
0	2	1
1	1	0
2	3	3
3	0	2

83. Using the accompanying graphs, find:

- a. $g(f(2))$ b. $f(g(-1))$ c. $g(f(0))$ d. $g(f(1))$



84. Given $F(x) = 2x + 1$ and $G(x) = \frac{x-1}{x+2}$, find:

- a. $F(G(1))$ d. $F(F(0))$
 b. $G(F(-2))$ e. $(F \circ G)(x)$
 c. $F(G(2))$ f. $(G \circ F)(x)$

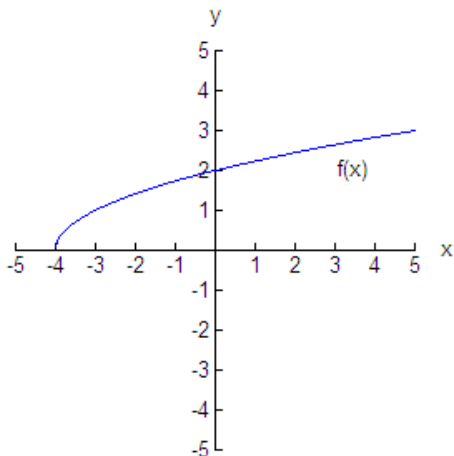
85. Find a new function $j(x)$ such that $j(x)$ is the composition of three functions f , g , and h : $j(x) = f(g(h(x)))$. Let $f(x) = 4x$, $g(x) = e^x$, and $h(x) = x - 1$.

86. Show that the functions are inverses of each other:

$$f(x) = \sqrt{x - 1} \text{ (where } x > 1) \text{ and } g(x) = x^2 + 1 \text{ (where } x > 0)$$

87. Find the inverse of the function: $f(x) = \sqrt[3]{4x + 5}$

88. Given the accompanying graph of the function $f(x)$, answer the following.



- Does $f(x)$ have an inverse? Please explain.
- What is the domain of $f(x)$? Estimate the range of $f(x)$.
- From the graph, determine $f(-4)$, $f(0)$, and $f(5)$.
- Determine $f^{-1}(0)$, $f^{-1}(2)$, and $f^{-1}(3)$.

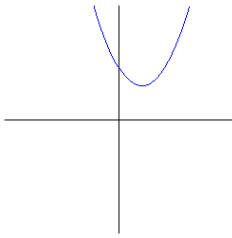
89. For the function Q , find Q^{-1} if it exists. If the inverse function exists, find $Q(3)$ and $Q^{-1}(3)$.

$$Q(x) = \frac{2}{3}x - 5$$

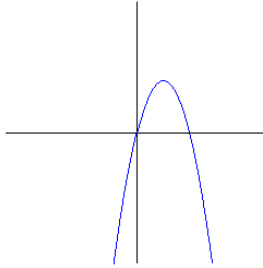
90. The formula for the volume of a cone is $V = \left(\frac{1}{3}\right)\pi r^2 h$. Assume you are holding a 6-inch-high sugar cone for ice cream.

- Construct a function $V(r)$ for the volume as a function of r . Find $V(1.5)$ and explain what you have found (using appropriate units).
- When dealing with abstract functions where $f(x) = y$, we have sometimes used the convention of using x (rather than y) as the input to the inverse function $f^{-1}(x)$. Explain why it does not make sense to interchange V and r here to find the inverse function.

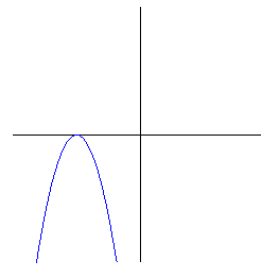
91. Given the graph of a quadratic function, $f(x) = ax^2 + bx + c$, what can you say about its discriminant, $b^2 - 4ac$?



a.

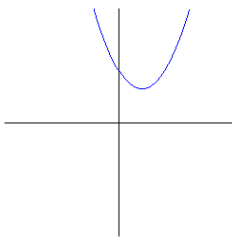


b.

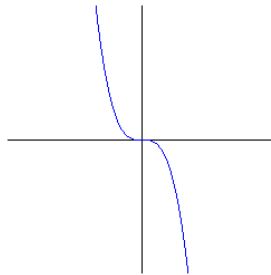


c.

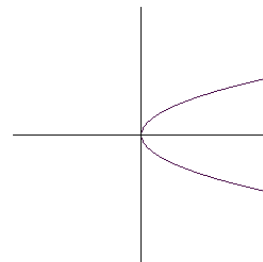
92. Which of the accompanying graphs describe one-to-one functions? Explain your answers.



a.



b.



c.

Solutions to Study Questions

MAC 1105, College Algebra Final Exam

1.
 - a. Function (all input values are different and thus each input has one and only one output)
 - b. Function (all input values are different and thus each input has one and only one output)
 - c. Not a function [same input values have different output values]
 - d. Not a function [same input values have different output values]

2.
 - a. Not a function: fails the vertical line test, e.g., look at the y-axis.
 - b. Function: passes the vertical line test.
 - c. Function; passes the vertical line test.

3. The equation is $C = 2.00 + 0.32M$. It represents a function. The independent value is M measured in miles. The dependant variable is C measured in dollars. Here is a table of values:

Miles	Cost (\$)	Miles	Cost (\$)
0	2.00	30	11.60
10	5.20	40	14.80
20	8.40	50	18.00

4.
 - a. $p(-4) = 0.063$, $p(5) = 32$ and $p(1) = 2$; b. $n = 1$ only

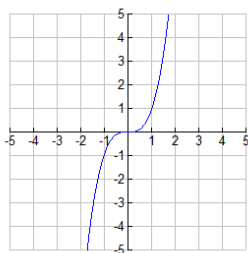
5.
 - a. $f(-2) = 5$, $f(-1) = 0$, $f(0) = -3$, and $f(1) = -4$
 - b. $f(x) = -3$ if and only if $x = 0$ or 2 .
 - c. $[-4, \infty)$, The range of f is from -4 to $+\infty$ including -4 , since we may assume that its arms are extended out indefinitely.

6.
 - a. $[-6, -3)$ and $(5, 11)$
 - b. $(-3, 5)$ and $(11, 12]$
 - c. $(-6, 2)$ and $(8, 12)$
 - d. $(2, 8)$
 - e. concave down over $(0, 5)$ and concave up over $(5, 8)$
 - f. $x = 1$ or $x = 3.5$
 - g. $f(-8)$ is undefined

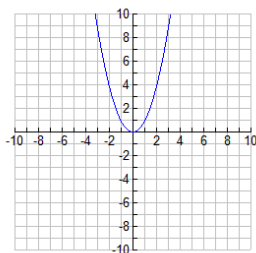
7.
 - a. Johnsonville's population goes from $2.4 \cdot 100,000 = 240,000$ to $5.8 \cdot 100,000 = 580,000$. Range for Johnsonville: $[240,000, 580,000]$; Palm City's population ranges from $1.8 \cdot 100,000 = 180,000$ to a high of $3.8 \cdot 100,000 = 380,000$. Range for Palm City: $[180,000, 380,000]$ (This notation adheres to what is found in the graph.)
 - b. The population of Palm City increased from 1900 to 1930.
 - c. The population of Palm City decreased from 1930 to 1990.
 - d. The two populations were equal sometime around 1940.

8. Here are the graphs asked for:

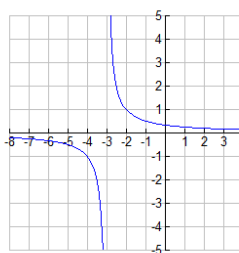
$$y_1 = x^3$$



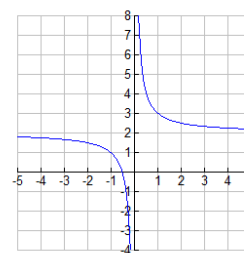
$$y_2 = x^2$$



$$y_3 = 1/(x + 3)$$



$$y_4 = 1/x + 2$$



- a. As $x \rightarrow +\infty$: y_1 approaches $+\infty$, y_2 approaches $+\infty$,
 y_3 approaches 0; y_4 approaches 2.
- b. As $x \rightarrow -\infty$: y_1 approaches $-\infty$, y_2 approaches $+\infty$,
 y_3 approaches 0, y_4 approaches 2.

9. a. $\frac{500,000 - 4,000,000}{2000 - 1930} = \frac{-3,500,000}{70} = -50,000$; The African elephant population is decreasing by an average of 50,000 elephants per year.

b. In 1980 the rate was twice as large as the rate computed in part (a) for 1930 to 2000. That means that either before or after the 1980's, the average rate of decline must have been much smaller.

10.

X	f(x)	Average Rate of Change
0	0	n.a.
1	1	1
2	8	7
3	27	19
4	64	37
5	125	61

- a. The function is increasing.
 b. The average rate of change is also increasing.

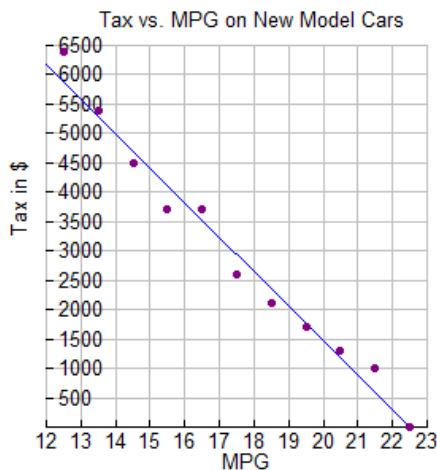
11. a. Graph C b. Graph A c. Graph B

12. a. slope of segment A = $[2 - (-6)]/[-4 - (-8)] = 8/4 = 2$
 slope of segment B = $[-8 - 2]/[0 - (-4)] = -10/4 = -2.5$
 slope of segment C = $[-8 - (-8)]/[2 - 0] = 0/2 = 0$
 slope of segment D = $[-6 - (-8)]/[6 - 2] = 2/4 = 0.5$
 slope of segment E = $[6 - (-6)]/[10 - 6] = 12/4 = 3$
 slope of segment F = $[-16 - 6]/[12 - 10] = -22/2 = -11$

b. The slope of the segment F is steepest.

c. Segment C has a slope equal to 0.

13. a. $1/10$
 b. $1/12$
 c. old: $\frac{3}{run} = \frac{1}{10} \Rightarrow run = 30$ ft; new: $\frac{3}{run} = \frac{1}{12} \Rightarrow run = 36$ ft.
14. a. $D = 3.40, 3.51, 3.62, 3.73,$ and $3.84,$ respectively
 b. 0.11 is the slope; it represents the average rate of change of the average consumer debt per year; it is measured in thousands of dollars per year.
15. a. matches Graph B; b. matches Graph D; c. matches Graph C; d. matches Graph A
16. a. Annual increase = $(32,000 - 26,000)/4 = \$1500.$
 b. $S(n) = 26,000 + 1500n,$ where $S(n)$ is measured in dollars and n in years from the start of employment.
 c. Here $0 \leq n \leq 20$ since the contract is for 20 years.
17. a. $V(t) \approx 70,000 + 26,000t,$ where $t =$ years since 1977. b. $V(33) = \$928,000.$
18. a. $y = 1400x - 2000$
 b. $y = 7x + 9$
 c. $y = .74x + 2.18$
 d. $y = 72$
 e. $x = 275$
 f. $y = \frac{1}{2}x - \frac{1}{2}$
19. a. Below are the graph of the data and a hand-drawn best-fit line.

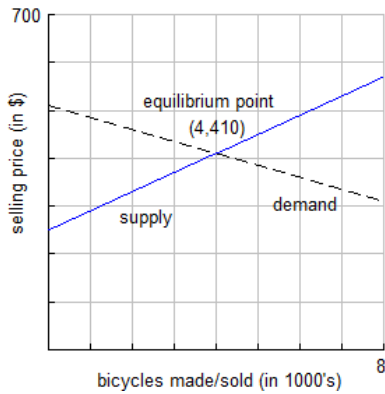


- b. The approximate coordinates of two points on this line are $(17.5, 3000)$ and $(22.5, 0) \Rightarrow$ the slope of this line is $\frac{0-3000}{22.5-17.5} = \frac{-3000}{5} = -600$ dollars per mpg. The equation of this line is $T(x) = -600x + 13500,$ where x is in mpg and $T(x)$ represents tax (in dollars). The vertical intercept is very large because its value is what one would get if mpg takes the value of 0. (This value, of course, represents an impossible situation.)

c The average rate of change for the given line is -600 dollars per mpg. As the mpg rating increases, the tax paid goes down by \$600 per mpg.

20. $y = -x - 2$ and $y = 2x - 8$ is the system, and the solution is $(2, -4)$. Check: $-2 - 2 = -4$ and $2 \cdot 2 - 8 = -4$, and thus the claimed solution works.

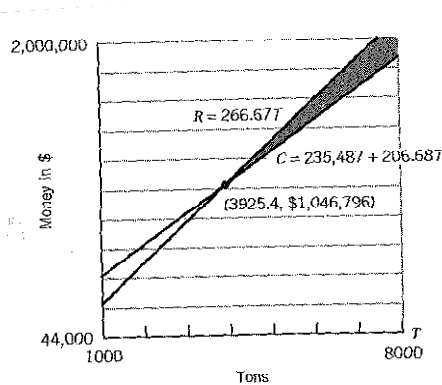
21. a. The graphs of the supply and demand equations are in the accompanying diagram.



b. The equilibrium point is shown in the diagram. It is the spot where supply meets the demand; i.e., if the company charges \$410 for a bike it will sell exactly 4000 of them and have none left over.

22. a. $C_{\text{conventional}}(n) = 10000 + 740n$ b. $C_{\text{solar}}(n) = 10150 + 540n$
 c. In 0.75 years, the costs would be the same, \$10555, for the two systems.
 d. conventional costs = \$13700 and solar costs = \$12850

23. a. $C = 235,487 + 206.68T$ gives the cost in dollars when T is measured in tons of fertilizer produced. $R = 266.67T$ gives the revenue in dollars from selling T tons of fertilizer.
 b. The graph is found in the accompanying diagram and the breakeven point is marked on the graph. It is where $T \approx 3925.4$ tons and $M \approx \$1,046,800$ dollars.



c. The profit region is where revenue is greater than cost. (see shading in above graph)

24. For Graph A: $x \geq 0$, $y \geq 0$, and $y < -1.5x + 3$ (This might also be written as $3x + 2y < 6$.)
 For Graph B: $y \geq x + 1$ and $y < 2x + 2$

25. a. $x - 5 = 3$ or $x = 8$
 b. $2x + 10 = 100$ or $2x = 90$ or $x = 45$
 c. $3x - 1 = -4$ or $3x = -3$ or $x = -1$
 d. $500 - 25x = 1000$ or $-25x = 500$ or $x = -20$

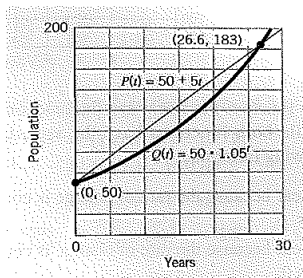
26. a. 1.17 b. 63 cells per ml.
 c.

d	C
0	63.0
1	73.7
2	86.2
3	100.9
4	118.1
5	138.11
6	161.6
7	189.1
8	221.2
9	258.8
10	302.8

- g. C doubles somewhere between $d = 4$ and $d = 5$ days.

27. a. $G(t) = (5 \cdot 10^3)(1.185)^t$ cells/ml, where t is measured in hours.
 b. $G(8) = (5 \cdot 10^3)(1.185)^8 = 19,440.92$ cell/ml

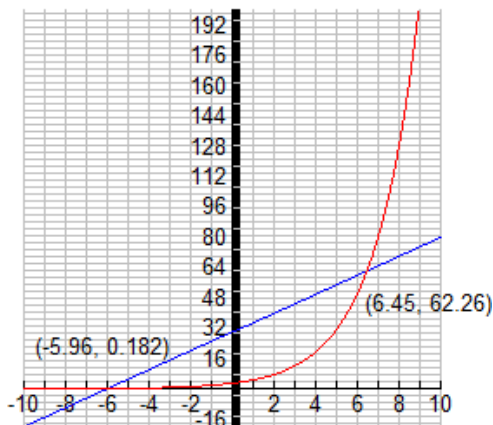
28. a. $P(t) = 50 + 5t$
 b. $Q(t) = 50(1.05)^t$
 c. The graph of the two functions is in the accompanying diagram.
 d. Graphing software gives that the two are equal at $(0, 50)$ and at $(26.6, 183)$. Thus the populations were both 50 people at the start and were both approximately 183 persons after 26.6 years. (Student eyeball estimates may differ.)



29. a. exponential: $f(x) = 7(2.5)^x$ b. linear: $g(x) = 0.2x + 0.5$

30. a. $f(t) = 10,000(0.4)^t$; b. $g(t) = (2.7 \cdot 10^{13})(0.27)^t$

31. a. $P(t) = 100b^t$; $99.2 = 100 \cdot b$ or $b = 0.992$ and therefore $P(t) = 100(0.992)^t$
 b. $P(50) = 100(0.992)^{50} \approx 66.9$ grams; $P(500) = 100(0.992)^{500} \approx 1.8$ grams
32. The function P goes with Graph C, the function Q goes with Graph A, the function R goes with Graph B, and the function S goes with Graph D.
33. The accompanying diagram contains the graphs of $f(x) = 30 + 5x$ and $g(x) = 3 \cdot 1.6^x$ with the points of intersection marked.
- $f(0) = 30$; $g(0) = 3$; $f(0) > g(0)$
 - $f(6) = 60$; $g(6) \approx 50.33$; $f(6) > g(6)$
 - $f(7) = 65$; $g(7) \approx 80.53$; $f(7) < g(7)$
 - $f(-5) = 5$; $g(-5) \approx 0.286$; $f(-5) > g(-5)$
 - $f(-6) = 0$; $g(-6) \approx 0.179$; $f(-6) < g(-6)$
 - f and g go to infinity; $f(x) < g(x)$ for all $x > 6.5$, approximately.
 - f goes to $-\infty$ and $g(x)$ goes to 0; thus $f(x) < g(x)$ for all $x < -6$, approximately.



34.

	Initial Value	Growth or Decay ?	Growth or Decay Factor	Growth or Decay Rate	Exponential Function
a.	600	Growth	2.06	106%	$y = 600(2.06)^t$
b.	1200	Growth	3.00	200%	$y = 1200(3)^t$
c.	6000	Decay	0.25	75%	$y = 6000(0.25)^t$
d.	$1.5 \cdot 10^6$	Decay	0.75	25%	$y = 1.5 \cdot 10^6 (0.75)^t$
e.	$1.5 \cdot 10^6$	Growth	1.25	25%	$y = (1.5 \cdot 10^6) (1.25)^t$
f.	7	Growth	4.35	335%	$y = 7(4.35)^t$
g.	60	Decay	0.35	65%	$y = 60(0.35)^t$

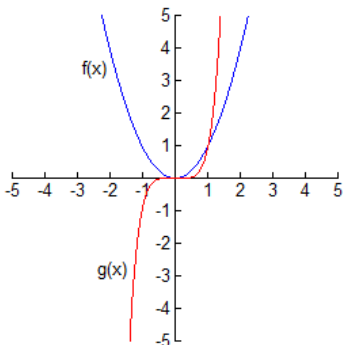
35. $A(x) = 500 \cdot 0.85^x$; decay rate of 15%

36. a. $P(t) = 150 \cdot 3^t$ b. $P(t) = 150 - 12t$ c. $P(t) = 150 \cdot 0.93^t$ d. $P(t) = 150 + t$

37. a. $A(t) = A_0(0.75)^t$, where t measures the number of years from the original dumping of the pollutant and A_0 represents the original amount of pollutant.
 b. We are solving $0.1 \cdot A_0 = A_0 (0.75)^t$ for t and we first divide by A_0 . Then $0.1 = 0.75^t$.
 $\log(0.1) = \log(0.75)^t$. $\log(0.1) = t \log(0.75)$. $\log(0.1)/\log(0.75) = t$. This gives $t \approx 8$ years.
38. a. Has a fixed doubling time. d. Has a fixed doubling time.
 b. Has neither e. Has neither
 c. Has a fixed half-life f. Has a fixed half-life
39. b. $f(t) = 1000(2)^{t/7}$; $2^{1/7} = 1.1041$ per year; 10.41% per year
 c. $f(t) = 4(2)^{t/25}$; $2^{1/25} = 1.0281$ per minute; 2.81% per minute
 d. $f(t) = 5000(2)^{t/18}$; $2^{1/18} = 1.0393$ per month; 3.935% per month
40. Eyeball estimates will vary. One set of guesses is:
 a. 1.3 hrs. b. 2.4 hrs. c. 4.4 hrs.
41. a. $\log\left(\frac{K^3}{(K+3)^2}\right)$ c. $\log(T^4 \cdot \sqrt{T}) = \log(\sqrt{T^9})$
 b. $\log\left(\frac{(3+n)^5}{m}\right)$ d. $\log\left(\sqrt[3]{xy^2}\right)$
42. a. 1000
 b. 999
 c. $10^{5/3} \approx 46.42$
 d. $1/9$
 e. No solution, since x cannot be negative.
43. a. $t = \log(2)/\log(1.5) \approx 1.71$ 20-minute time periods or about 34 min.
 b. $t = \log(10)/\log(1.5) \approx 5.68$ 20-minute time periods or about 114 min.
44. a. $B(t) = B_0(1/2)^{t/20}$, where t is measured in minutes and $B(t)$ and B_0 are measured in some weight unit. None is specified in the problem.
 b. In one hour, $t = 60$, so $B(60) = B_0(1/2)^{60/20} = 0.125B_0$ or 12.5% of original amount left.
 c. If its half-life is 20 minutes, then its quarter-life is 40 minutes.
 d. Solving $0.10 = \left(\frac{1}{2}\right)^{t/20}$ for t , we get $\log(0.10) = (t/20) \log(1/2) \dots 20 \log(0.10)/\log(1/2) = t \dots$
 $t =$ about 66 minutes
45. a. $A(t) = 25,000(1 + .0575/4)^{4t}$, where $t =$ number of years
 b. $B(t) = 25,000 \cdot e^{0.0575t}$, where $t =$ number of years
 c. $A(5) = \$33,259.12$ and $B(5) = \$33,327.26$

- d. The effective rate for compounding quarterly is $(1 + \frac{0.0575}{4})^4 - 1 \approx 0.05875$, and the effective rate for compounding continuously is $e^{0.0575} - 1 \approx 0.05919$. Thus continuous compounding has a slightly greater effective rate.
46. a. $U(x) = 10e^{-0.139x}$, where x is measured in billions of years.
b. $5 = 10e^{-0.139x}$, and thus $\frac{1}{2} = e^{-0.139x}$, $\Rightarrow \ln(1/2) = -.139x$, $\Rightarrow x = 4.987$, about 5 billion years.
47. a. $\frac{1}{2}(\ln 4 + \ln x + \ln y)$ b. $\frac{1}{3}(\ln 2 + \ln x) - \ln(4)$ c. $\ln(3) + \frac{3}{4} \ln(x)$
48. a. $x = \ln(10) \approx 2.303$ b. $x = \log(3) \approx 0.477$ c. $x = \log(5)/\log(4) \approx 1.161$
d. $x = e^5 \approx 148.413$ e. $x = e^3 - 1 \approx 19.086$ f. No solution [$\ln(-4/3)$ not defined]
49. $y = Ce^{-0.07t}$, and thus the effective interest rate $r = e^{0.07} - 1 = 0.0725 = 7.25\%$.
50. a. $A = \log(x)$, $B = \log(x-1)$, and $C = \log(x-2)$; so $f(x)$ matches A, $g(x)$ matches B, and $h(x)$ matches C.
b. $\log(1) = 0$; thus $f(1) = g(2) = h(3) = 0$.
c. f has 1 as its x -intercept; g has 2 and h has 3.
51. a. $N = 10 e^{(\ln 1.045)t} = e^{0.0440t}$ b. $Q = 5 \cdot 10^{-7} \cdot e^{(\ln 0.072)A} = 5 \cdot 10^{-7} \cdot e^{-2.631A}$
c. $P = 500 \cdot e^{(\ln 2.10)x} = 500 \cdot e^{0.742x}$
52. a. 35 micrograms at $t = 0$; b. $A(7) = 28.57$ micrograms; c. $t = 122.6$ days; d. $t = 23.90$ days
53. a. Since $Q(75000) = 0.5Q_0$, the half-life is 75000 years.
b. $Q = Q_0(.99999)^t$
c. The annual percent change is $.99999 - 1 = -0.00001 = -0.001\%$, so annual decay rate = 0.001%
54. a. $P(t) = 12e^{rt} \rightarrow 600,000,000 = 12e^{r(90)} \rightarrow 50,000,000 = e^{90r} \rightarrow \ln(50,000,000) = 90r \rightarrow r = .1970$
 $\rightarrow P(t) = 12e^{0.1970t}$; b. $P(141) = 1.389 \times 10^{13}$ or almost 14 trillion rabbits
55. a. S is multiplied by 16; V is multiplied by 64.
b. S is multiplied by n^2 ; V is multiplied by n^3 .
c. S is divided by 9; V is divided by 27.
d. S is divided by n^2 ; V is divided by n^3 .
56. a. $Y = kX^3$ b. $10 = k(2)^3 \rightarrow k = 10/8 = 1.25$
c. $k(5X)^3 = 125kX^3$, so Y is multiplied by a factor of 125
57. $L = kx^5 \rightarrow L = k(2x)^5 \rightarrow L = 32kx$ so L is multiplied by 32 or is 32 times greater
58. a. $Q = k\sqrt[3]{t} d^2$, $k = 1$; b. $A = khr^2$, $k = 2\pi$; c. $V = kBh$, $k = 1$; d. $T = k\sqrt{p} u^2$, $k = 0.25$

59. a. $f(0) = g(0) = 0$ b. If $0 < x < 1$, then $f(x) > g(x)$
 c. $f(1) = g(1) = 1$ d. If $x > 1$, then $f(x) < g(x)$
 e. If $x < 0$, then $f(x) > g(x)$



60. a. n is even; b. $k < 0$; c. Yes, $f(-2) = f(2)$; d. Yes, $f(-x) = f(x)$; e. $-\infty$; f. $-\infty$

61. a. $f(x) > g(x)$ as $x \rightarrow -\infty$
 b. $f(x) < g(x)$ as $x \rightarrow +\infty$
 c. $f(x) > g(x)$ for x in $(-\infty, -1)$
 d. $f(x) < g(x)$ for x in $(-0.5, 1)$
 e. $f(x) > g(x)$ for x in $(1, 6)$
 f. $g(x) > f(x)$ for x in $(6, +\infty)$, therefore definitely $>$ in $(7, +\infty)$

62. Linear function: $m = \frac{32-0.5}{4-1} = 10.5 \Rightarrow y = 10.5x - 10$.
 Exponential function: $y = \frac{1}{8} \cdot 4^x$
 Power function: $y = 0.5x^3$

63. Given $I(x) = k/x^2$ and $I(6) = 4$, then $4 = k/36$; thus $k = 144 \Rightarrow I(8) = 144/64 = 2.25$ watts per square meter and $I(100) = 144/10000 = 0.0144$ watt per square meter.

64. a. The volume becomes $1/3$ of what it was.
 b. The volume becomes $1/n$ of what it was.
 c. The volume is doubled.
 d. The volume becomes n times what it was.

65. a. n is even b. $k < 0$ c. $f(-1) < 0$ d. yes e. 0 f. 0

66. a. Graphs A & C; b. Graphs B & D; c. Graphs A & C; d. Graphs B & D; e. Graphs B & C

67. a. concave up; minimum, vertex at $(-1, -4)$; the line $x = -1$ is axis of symmetry; $(1,0)$ and $(-3,0)$ are horizontal intercepts; $(0,-3)$ is vertical intercept.
 b. concave down; maximum; vertex at $(2,9)$; the line $x=2$ is axis of symmetry; $(-4,0)$ and $(8,0)$ are horizontal intercepts; $(0,8)$ is vertical intercept.
 c. concave down; maximum; vertex at $(-5,3)$ the line $x = -5$ is axis of symmetry; $(-8, 0)$ and $(-2,0)$ are horizontal intercepts; $(0,-5)$ is vertical intercept. [Students may have a different y value for vertical intercept.]

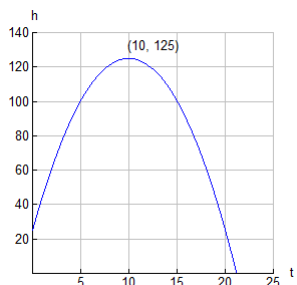
68. a. $(0,0)$; minimum; b. $(0,0)$, minimum; c. $(0,0)$, maximum; d. $(0,0)$, maximum

69. a. $(3,5)$; maximum b. $(-1, 8)$; minimum c. $(-4, -7)$; maximum d. $(2, -6)$; minimum

70. Graph A: $y = -2(x + 1)^2 + 4$; Graph B: $y = 10/9(x + 3)^2 + 4$

71. The maximum profit occurs when $x = -20/[2 \cdot (-0.5)] = 20$, meaning that \$20,000 must be spent; the maximum profit is 430 thousand dollars.

72. a. $(0,25)$; The initial height of the object is 25 ft. b. $(10,125)$; c. See graph below

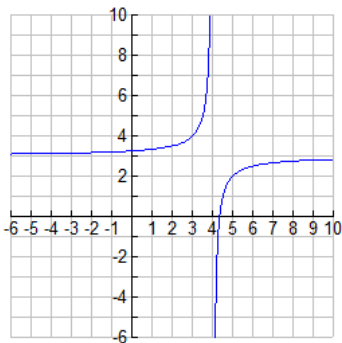


- d. The object reaches its maximum height of 125 ft, 10 seconds after being thrown upwards from the surface of the planet.; e. It makes sense for $[0, 21.18]$.

73. a. y-intercept is $(0, -1)$; $y = (3x - 1)(x + 1)$ and thus the x-intercepts are $(1/3, 0)$ and $(-1, 0)$.
 b. y-intercept is $(0,11)$; the x-intercepts are $(\frac{6 \pm \sqrt{3}}{3}, 0)$ or $\approx (2.58, 0)$ and $(1.42, 0)$.
 c. y-intercept is $(0, 15)$; x-intercepts are $(5/2, 0)$ and $(-3/5, 0)$.
 d. The vertical intercept is $(0, -5)$ and the x-intercepts are $(\pm\sqrt{5}, 0)$.
 e. The vertical intercept is $(0, 1)$ and there are no x-intercepts since all the zeros of $f(x)$ are complex with an imaginary part.

74. a. Discriminant = $49 > 0$, therefore predict two horizontal intercepts; $(1,0)$ and $(-2.5, 0)$
 b. Discriminant = 0 , therefore predict one horizontal intercept; $(4,0)$
 c. Discriminant = $-4 < 0$, therefore predict no horizontal intercepts
 d. Discriminant = $-8 < 0$, therefore predict no horizontal intercepts
 e. Discriminant = $29 > 0$, therefore predict two horizontal intercepts; $(\frac{3 \pm \sqrt{29}}{-2}, 0)$

75. a. goes with Graph A; b. goes with Graph C; c. goes with Graph B
76. For Graph A: a. 2 b. 2 c. minus d. 3
 For Graph B: a. 3 b. 2 c. minus d. 4
 For Graph C: a. 4 b. 5 c. plus d. 5
77. Answers will vary: a. $y = (x - 4)(x + 5)(x + 6)$; b. $y = (x + 1)(x - 3)(x - 10)$; c. $y = 2x^3 - 5x + 4$;
 d. $y = x^5 - 5x - 4$
78. a. True; it a function and 0 is in its domain.
 b. True; it is a cubic.
 c. True. The range is all real numbers, therefore 0 is in its range.
 d. False; it is d and d can be positive, negative, or 0.
 e. True, since the leading coefficient is positive.
 f. True, since the degree is three.
79. a. $j(x) = 3x^5 + x^2 + x - 1$; $k(x) = 3x^5 - x^2 + x + 1$; $m(x) = 3x^7 - 3x^5 + x^3 - x$
 b. $j(2) = 101$; $k(3) = 724$; $m(-1) = 0$
80. a. -6; b. -16; c. -315; d. 5
81. Horizontal asymptote: $y = 3$; Vertical asymptote: $x = 4$



82. a. $f(g(1)) = f(0) = 2$; b. $g(f(1)) = g(1) = 0$; c. $f(g(0)) = f(1) = 1$; d. $g(f(0)) = g(2) = 3$; e. $f(f(2)) = f(3) = 0$
83. a. $g(f(2)) = g(0) = 1$; b. $f(g(-1)) = f(2) = 0$; c. $g(f(0)) = g(4) = -3$; d. $g(f(1)) = g(3) = -2$
84. a. $F(G(1)) = F(0) = 1$ b. $G(F(-2)) = G(-3) = 4$
 c. $F(G(2)) = F(0.25) = 1.5$ d. $F(F(0)) = F(1) = 3$
 e. $(F \circ G)(x) = 2 \left(\frac{x-1}{x+2} \right) + 1 = \frac{(2x-2) + (x+2)}{x+2} = \frac{3x}{x+2}$
 f. $(G \circ F)(x) = \frac{(2x+1)-1}{(2x+1)+2} = \frac{2x}{2x+3}$

85. $j(x) = 4e^{x-1}$
86. $f(g(x)) = f(x^2 + 1) = \sqrt{(x^2 + 1) - 1} = \sqrt{x^2} = x$, since $x > 0$
 $g(f(x)) = g(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = (x-1) + 1 = x$
87. $f(x) = \sqrt[3]{4x+5}$ implies $y = \sqrt[3]{4x+5}$; solve for x : $y^3 = 4x+5$, $y^3 - 5 = 4x$, $\frac{y^3-5}{4} = x$; exchange x & y since this is a general equation where x & y do not have specific meaning: $\frac{x^3-5}{4} = y$,
 $f^{-1}(x) = \frac{x^3-5}{4}$
88. a. Yes, $f(x)$ has an inverse since its graph passes the horizontal line test.
 b. The domain of $f(x)$ is the interval $[-4, \infty)$. The range of $f(x)$ is the interval $[0, \infty)$.
 c. $f(-4) = 0$, $f(0) = 2$, and $f(5) = 3$. This means that the points $(-4, 0)$, $(0, 2)$, and $(5, 3)$ all lie on the graph of $f(x)$.
 d. Given the results in part (c), the points $(0, -4)$, $(2, 0)$, and $(3, 5)$ all lie on the graph of f^{-1} . Thus, $f^{-1}(0) = -4$, $f^{-1}(2) = 0$, $f^{-1}(3) = 5$.
89. $Q^{-1}(x) = \frac{3x+15}{2}$, $Q(3) = -3$, $Q^{-1}(3) = 12$
90. a. $V(r) = 2\pi r^2$. $V(1.5) = 14.14$ cubic inches. This is the amount of ice cream the cone itself can hold when the radius of the top of the cone is 1.5 inches.
 b. In most scientific equations where the variables have physical meanings, as here, you cannot switch v (the volume) for r (the radius), as it would result in an incorrect formula.
91. a. $b^2 - 4ac < 0$; Since there are no x -intercepts, the zeros of the function must be complex with an imaginary part.
 b. $b^2 - 4ac > 0$; Since there are two x -intercepts, the zeros of the function must be real.
 c. $b^2 - 4ac = 0$; Since there is one x -intercepts, the zero of the function is a repeated real number.
92. Only graph b is a one to one function, since it passes both the vertical and horizontal line tests.