MAS2103 Course Outline

Linear Algebra.....(4) (P)

Description: This is a one semester introductory course in linear algebra. This course includes, but is not limited to, the study of systems of linear equations, matrices, determinants, vectors and vector spaces, linear transformations, eigenvalues and eigenvectors, and their applications. Computer software and graphing calculators will be used to enhance the learning and teaching of topics and techniques covered.

General Education Learning Outcome: The primary General Education Learning Outcome (GELO) for this course is Quantitative Reasoning, which is to understand and apply mathematical concepts and reasoning, and analyze and interpret various types of data. The GELO will be assessed through targeted questions on either the comprehensive final or an outside assignment.

Prerequisite: MAC2312 with a grade of "C" or better, OR the equivalent.

Rationale: In an increasingly complex world, mathematical thinking, understanding, and skill are more important than ever. MAS2103 will show students how to simplify many types of complex problems using matrix algebra and vector geometry. Students who major in the sciences or engineering are often required to study linear algebra. This course provides a solid foundation for further study in mathematics, the sciences, and engineering.

Impact Assessment: *Linear Algebra* plays a significant role in many areas of mathematics, statistics, engineering, the natural sciences, and the computer sciences. Students who major in these fields will need some familiarity with linear algebra and its applications. The course applies toward the General Education mathematics requirement area B for an Associate of Arts degree. *Introduction to Linear Algebra* is a terminal course. It is not a prerequisite for any course, but it provides a foundation of important mathematical ideas that will help students be successful in future coursework.

Broad Course Objectives: This course supports the following goals of the Math Department:

- Engage students in sound mathematical thinking and reasoning. This should include students finding patterns, generalizing, and asking/answering relevant questions.
- Provide a setting that prepares students to read and learn mathematics on their own.
- Explore multiple representations of topics including graphical, symbolic, numerical, oral, and written. Encourage students to make connections among the various representations to gain a richer, more flexible understanding of each concept.
- Analyze the structure of real-world problems and plan solution strategies. Solve the problems using appropriate tools.
- Develop a mathematical vocabulary by expressing mathematical ideas orally and in writing.
- Enhance and reinforce the student's understanding of concepts through the use of technology when appropriate.

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As a result of successfully completing MAS2103, students should be able to demonstrate the following:

- Analyze/interpret quantitative data verbally, graphically, symbolically and numerically.
- Communicate quantitative data verbally, graphically, symbolically and numerically.
- Appropriately integrate technology into mathematical processes.
- Use mathematical concepts in problem-solving through integration of new material and modeling.

Topical Outline with Specific Course Objectives:

I. *Systems of Linear Equations*

A. Recognize a linear equation in n variables.

B. Find a parametric representation of a solution set of a system of linear equations.

C. Write a system of linear equations in matrix form.

D. Solve a system of linear equations by substitution, graphing, using a computer or graphing calculator, Gaussian elimination, Gauss-Jordan elimination, LU-factorization, Cramer's Rule.

E. Determine whether a system of linear equations is consistent or inconsistent.

F. Write a given system of linear equations in the form $\mathbf{Ax} = \mathbf{b}$ and use it to solve for $\mathbf{x}$.

G. Find a general solution of a consistent system.

II. *Matrices*

A. Determine the size, transpose, inverse, rank, and LU-factorization of a matrix.

B. Write an augmented or coefficient matrix from a system of linear equations.

C. Use elementary row operations to write a matrix in row-echelon form and reduced row-echelon form.

D. Perform matrix operations and solve matrix equations.

E. Factor a given matrix into a product of elementary matrices.

F. Find the adjoint of a matrix and use it to find the inverse of the matrix.

G. Know and use equivalent conditions for an invertible matrix.

III. *Determinants*

A. Find the determinant, minors and cofactors of a given matrix.

B. Use the determinent to decide whether a given matrix is singular or nonsingular.

C. Use properties of determinants.

IV. Vectors and Vector Spaces

A. Perform vector operations for vectors in R^n.

B. Determine whether a given set with two operations is a vector space.

C. Recognize standard examples of vector spaces: n^{th} dimensional Euclidean space, the set of all $m \times n$ matrices, the set of all polynomials, the set of all polynomials of degree $\leq n$, the set of all continuous functions defined on the real line, the set of all continuous functions defined on a closed interval $[a,b]$.

D. Determine whether a given subset of a vector space is a subspace .

E. Recognize subspaces of R^2 and R^3 and understand their geometric interpretations.

F. Determine whether a vector is a linear combination of a given finite set of vectors in a vector space and be able to write this linear combination.

G. Determine whether a given set of vectors in a vector space is a spanning set for that vector space.

H. Determine whether a given finite set of vectors in a vector space is linearly independent.

I. Determine whether a given set of vectors in a vector space forms a basis for that vector space and recognize standard bases in the vector spases n^{th} dimensional Euclidean space, the set of all $m \times n$ matrices, and the set of all polynomials of degree $\leq n$.

J. Find the dimension of a subspace.

K. Find a basis for the column or row space of a matrix.

L. Find a basis for and the dimension of the nullspace of a matrix.

M. Find the coordinate matrix for a vector relative to a basis.

N. Find the transition matrix from one basis to another basis.

O. For a given vector $\mathbf{v}$, find its length, a unit vector in the same or opposite direction, all vectors that are orthogonal to $\mathbf{v}$, and the projection of $\mathbf{v}$ onto a given vector or vector space.

P. Find the distance, the dot product, the inner product, the cross product and the angle between any two given vectors in a Eulcidean space.

Q. Verify and use the Cauchy-Schwarz Inequality, the Triangle Inequality and the Pythagorean Theorem for vectors.

R. Determine whether any two given vectors are orthogonal, parallel, or neither.

S. Determine whether a given set of vectors is orthogonal, orthonormal, or neither.

T. Determine whether given subspaces are are orthogonal.

U. Find the orthogonal complement of a given subspace.

V. Apply the Gram-Schmidt orthonormalization process to find an orthonormal basis for a given basis, subspace, or inner product space.

W. Find an orthonormal basis for the solution space of a homogeneous system of linear equations.

V. Linear Transformations

A. Find the image and preimage of a given function.

B. Determine whether a function from one vector space to another is a linear transformation.

C. For a given linear transformation, find the kernel and range, find the basis for the kernel and range, and determine the nullity and rank.

D. Determine whether a given linear transformation is one-to-one or onto.

E. Determine whether two vector spaces are isomorphic.

F. Find the standard matrix for a given linear transformation and the composition of linear transformations.

G. Determine whether a given linear transformation is invertible and find its inverse if it exists.

H. Know and use the properties of similar matrices.

VI. Eigenvalues and Eigenvectors

A. Verify an eigenvalue and an eigenvector of a given matrix.

B. Understand the geometrical interpretation of the eigenvalue and eigenvector of a given matrix.

C. Find the characteristic equation and the eigenvalues and corresponding eigenvectors of a given matrix.

D. Determine whether a given matrix is diagonalizable, symmetric, or orthogonal.

E. Find (if possible) a nonsingular matrix P for a given matrix A such that $P^{-1}AP$ is diagonal.

F. Find a basis B (if possible)for the domain of a linear transformation T such that the matrix of T relative to B is diagonal.

G. Find the eigenvalus of a given symmetric matrix and determine the dimension of the corresponding eigen space.

H. Find an orthogonal matrix that diagonalizes a given matrix.

VII. Applications

A. Applications of the above objectives to geometry and other topics which could include differential equations, systems of linear differential equations, least squares approximations, Fourier approximations, population growth or models in dynamical systems.

Evaluation: Each instructor will determine the specific criteria for determining the final course grade. These criteria will be delineated in the first day handout provided to each student. Each instructor will give a comprehensive final exam during the assigned final exam period.

Commonality: All instructors will use the same textbook and cover all topics in the topical outline. A computer lab with mathematical software is provided to facilitate collaboration and the use of technology. A graphing calculator will be required for this course. Either the TI-83 or the TI-84 line of calculators is recommended; any other graphing calculator will need to be approved by the instructor.